

QCD

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Abstract

This is the written version of the lecture on deep inelastic scattering and related topics in QCD, delivered in the course of the XXVI International Meeting on Fundamental Physics to an audience of young experimentalists. The aim is fundamentally pedagogical. I review the theoretical setting of the Altarelli-Parisi equations, discuss recent determinations of α_s from deep-inelastic scattering and then move to the kinematical region explored by HERA. In the way I mention some unsolved theoretical problems. I discuss low- x physics and to what extent $\log \frac{1}{x}$ resummations are called for.

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1 Introduction

These notes are not an introduction to Quantum Chromodynamics (QCD), the theory of strong interactions. Many excellent textbooks exist where the interested reader can find clear expositions of the subject[1]. In fact we shall assume here that the reader is familiar with the basic technical tenets of perturbative QCD, such as Feynman diagrams, dimensional regularization, renormalization, beta function and so on. We pretend rather to give physical insight of the reasons behind a rather remarkable theoretical development which is nearly as old as QCD itself, namely deep inelastic scattering. Why is it so remarkable?

QCD is, in a way, a rather simple theory (specially when compared to the intricacies of the electroweak part of the Standard Model). It is just a simple extension of good old Quantum Electrodynamics. Instead of matter fields carrying electrical charge $+1$ (and anticharge -1), if we are talking about electrons, the matter fields of QCD (the quarks) carry a new quantum number: color. Color can take three different values (and their corresponding anti-values). Furthermore the intermediate bosons (the gluons), unlike photons which cannot change the charge of a particle, change the color of a quark. They may for instance turn a red quark into a blue quark. This simple fact implies that the gauge group of QCD is much larger than the $U(1)$ of QED. Since every quark comes in three copies, whose labels get exchanged, there is a $SU(3)$ invariance¹.

Simple as this theory may seem, it is not an easy matter in QCD to relate theory and experiment. It is well known that the fields and particles we know how to compute with (with the simplest tool at our disposal, perturbation theory) are not those that are observed by experimentalists in their detectors due to the phenomenon of confinement. Quarks and gluons are real, but they cannot be detected as free particles as they are known to be confined inside hadrons. In view of this is quite remarkable that there are theoretical techniques enabling us to put the theory to very stringent tests.

The phenomenon of confinement comes about because the coupling constant of QCD (which is relatively small at large values of the momentum transfer, the value quoted by the Particle Data Group[2] is $\alpha_s(M_Z) = 0.119 \pm 0.002$; notice the truly amazing precision, which may soon be reduced to a mere 1%) becomes strong as the energy decreases. The behaviour

¹The reason why the symmetry group is $SU(3)$ and not $U(3)$ —which would have nine gluons— has to do with the mathematical fact that $\epsilon_{\alpha\beta\gamma}$ is not an invariant tensor of $U(3)$. This tensor is necessary to build combinations of three quarks which are antisymmetric, as required by Fermi statistics. For instance, $\Delta^{++} = |u^\dagger u^\dagger u^\dagger\rangle$. Being the lightest hadron with this quark contents we expect to have the three quarks in the ground state, hence in a symmetric wave function. This is in contradiction with Fermi statistics. The contradiction can be solved if we admit the existence of a new quantum number α and $|u^\dagger u^\dagger u^\dagger\rangle = \frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}|u^\dagger_\alpha u^\dagger_\beta u^\dagger_\gamma\rangle$.

predicted by the renormalization-group is, at one loop,

$$\alpha_s(Q) = \frac{-\pi}{\frac{\beta_1}{2} \log(Q^2/\Lambda_{QCD}^2)}. \quad (1)$$

where Λ_{QCD} is a renormalization-group invariant, but scheme dependent, quantity and $\beta_1 = -11/2 + N_f/3$. The preferred value is $\Lambda_{QCD} = 219_{-23}^{+25}$ MeV (5 flavours, \overline{MS} -scheme). The meaning of the scale-dependent, renormalized coupling constant is roughly the following: it is the “effective” coupling, relevant at the scale Q , namely, the one that (within the chosen scheme) minimizes further quantum corrections, in particular resumming all large logs. From (1) we see that precisely at the scale $Q^2 = \Lambda_{QCD}^2$, the effective coupling has a pole. Of course well before that scale is reached perturbation theory becomes completely unreliable, and the $1/r$ potential of the perturbative interaction change to a stronger behaviour, possibly to a linear $\sim r$ behaviour. When computing the production of any physical hadron (typically of mass \sim few Λ_{QCD}), perturbation theory is completely useless.

One instance where perturbative QCD can be applied is to inclusive processes, provided that the characteristic momentum transfer is large enough. These will not be discussed in this lecture. The interested reader can look at the determination of α_s through R_{had} R_τ , for instance in.[3]

A clear application of perturbative QCD is provided by deep inelastic scattering. The subject is now over twenty years old and, by now, perturbative QCD has been tested to a high degree. Furthermore, the commissioning of HERA has opened a new kinematical region where it may be possible to study the onset of non-perturbative effects in a controlled fashion. The exploration of this region is a fascinating subject interesting on its own right.

Due to the lack of time and space we have not included two sections that, in our view, should be in any general review of perturbative QCD and deep inelastic scattering. The first one concerns the so-called “spin of the proton” problem[4]. Another topic that is not covered at all is the study of the photon structure functions.[5] The list of references is very incomplete and those provided merely reflect personal tastes.

2 Logs in QCD

Beyond tree level most Feynman diagrams are ultraviolet divergent. Take for instance the one diagram contributing at one loop to the gluon propagator. Neglecting external momenta, the integral over the momenta of the internal particles is of the form

$$\int \frac{d^4k}{(2\pi)^4} \frac{k^\alpha k^\beta}{k^4} = \infty. \quad (2)$$

To make sense of the theory and get a finite result we must introduce a cut-off Λ and counterterms. A possible method is to perform a subtraction at some $q^2 = -\mu^2$. For instance, for the self-energy of the gluon propagator

$$\Pi(q^2) - \Pi(-\mu^2) \equiv \Pi_R(q^2) = \text{finite}. \quad (3)$$

Alternatively we can make sense of the integrals using dimensional regularization by continuing the dimensionality from 4 to $n = 4 + 2\epsilon$,

$$\int \frac{d^4 k}{(2\pi)^4} \rightarrow \int \frac{d^n k}{(2\pi)^n}, \quad (4)$$

and subtract just the poles in $1/\epsilon$ (minimal subtraction, MS) or also the $\gamma_E - \log 4\pi$ that always accompanies the singularity in $1/\epsilon$ (improved minimal subtraction, \overline{MS}). For instance, for the quark contribution to the gluon self-energy, one has the following result after computing the integral in $n = 4 + 2\epsilon$ dimensions

$$\Pi(q^2) = -\frac{\alpha_s}{6\pi} \delta_{ab} \left(\frac{1}{\epsilon} + \gamma_E + \log \frac{m^2}{4\pi\mu^2} + \dots \right). \quad (5)$$

Using the MS and \overline{MS} schemes one gets

$$\Pi_{MS}(q^2) = -\frac{\alpha_s}{6\pi} \delta_{ab} (\gamma_E + \log \frac{m^2}{4\pi\mu^2} + \dots) \quad \Pi_{\overline{MS}}(q^2) = -\frac{\alpha_s}{6\pi} \delta_{ab} (\log \frac{m^2}{\mu^2} + \dots) \quad (6)$$

The above expressions illustrate the appearance of ultraviolet logs $\log q^2/\mu^2$ through the renormalization procedure. There are, however, other source of logs in QCD. They are of the form $\log q^2/\lambda^2$ where λ^2 can be some external momentum squared or a small (mass)² that we have given by hand to the (massless) gluon. While the former are associated to ultraviolet divergent integrals (integrals with a bad behaviour when the internal momentum is large), the latter are infrared logs and are related to Feynman diagrams with a bad behaviour when one or more external momenta vanish. Ultraviolet logs appear in any renormalizable field theory after renormalization. On the contrary, infrared logs appear whenever a theory has massless particles in the spectrum (such as photons or gluons). A given Feynman diagram can give rise to both type of singularities at the same time.

There are actually two classes of infrared logs caused by massless particles. The so-called infrared divergences arise from the presence of a *soft massless* particle ($k^\mu \rightarrow 0$). For instance in the process $e^+e^- \rightarrow \mu^+\mu^-$ at the one loop level we have to compute the integral (figure 1)

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 [(p_1 + k)^2 - m^2] [(p_2 + k)^2 - m^2]}. \quad (7)$$



Figure 1: Example of diagram leading to an infrared divergence.

When $p_1^2 = p_2^2 = m^2$ the integral behaves for $k^\mu \rightarrow 0$ as

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4}. \quad (8)$$

and diverges. This divergence is unphysical so it must be cancelled by something else. The Bloch-Nordsieck theorem[6] states that in inclusive enough cross-sections the infrared logs cancel. What do we mean by ‘inclusive enough’? A detector will not be able to discern a ‘true’ muon from a muon accompanied by a soft enough photon (with $\vec{k} \rightarrow 0$). Therefore, we have to consider diagrams where a soft photon is radiated by the muon, square the modulus of the amplitude and integrate over the available phase space (which actually depends on the experimental cut). When this is done the result is infrared finite. The relevant diagrams are depicted in figure 2.

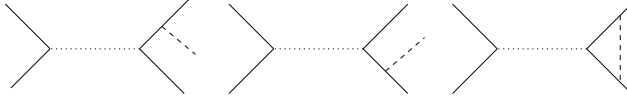


Figure 2: Real and virtual photons have to be included for IR safe results.

The other type of infrared logs are called mass singularities. They occur in theories with massless particles because two *parallel massless* particles have an invariant mass equal to zero

$$k^2 = (k_1 + k_2)^2 = \|(\omega_1 + \omega_2, 0, 0, \omega_1 + \omega_2)\| = 0. \quad (9)$$

The appearance of such a mass singularity is illustrated in figure 3

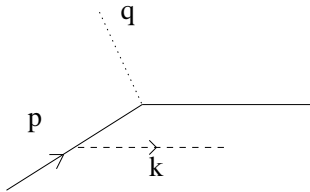


Figure 3: Diagram with a mass singularity.

$$\frac{1}{(p-k)^2} = \frac{1}{p^2 + k^2 - 2k^0 p^0 + 2k^0 p^0 \cos \theta}, \quad (10)$$

the denominator vanishes when we set all particles on shell ($p^2 = k^2 = 0$) and $\theta \rightarrow 0$ (i.e. \vec{k} is parallel to \vec{p}). Even if one of the two particles is massive there is a singularity, provided the 3-momenta are parallel.

The Kinoshita-Lee-Nauenberg theorem[7] ensures that for inclusive enough cross section the mass singularities also cancel. Both for mass singularities and for infrared divergences there is a trade-off between λ^2 , the infrared regulator of a massless particle, and the energy and angle resolution of the inclusive cross section $\Delta E, \Delta\theta$.

In practice, it is better to regulate the infrared logs using dimensional regularization (introducing λ^2 leads to difficulties with gauge invariance). Real gluon emission diagrams are regulated by performing the phase space integration in n dimensions.

There is in fact a lot of physical insight hidden in the infrared logs. Physical arguments tell us that the probability of finding a ‘bare’ isolated muon should be *zero*. We know this because detectors are unable to tell apart a muon from a muon plus one soft photon or indeed from a muon plus any number of soft photons. Infrared divergences in QED can be summed up and then one sees that the probability of finding an isolated muon is indeed zero and not infinite as the one loop diagram led us to believe. Whenever a Feynman diagram is infrared divergent it means that we have forgotten something relevant.

Let us consider in QED the interaction of a charged fermion with an external source and let us expand in the number of *virtual* photons n (i.e. in the number of loops). The total amplitude will be expressed as

$$M(p, p') = \sum_{n=0}^{\infty} M_n(p, p') \quad (11)$$

then a calculation shows that

$$\begin{aligned} M_0 &= m_0, \\ M_1 &= m_0 \alpha B + m_1, \\ M_2 &= m_0 \frac{(\alpha B)^2}{2} + m_1 \alpha B + m_2, \\ &\dots \end{aligned} \quad (12)$$

The quantities m_n are IR-finite, while B is IR-divergent. The series in (11) can be summed up

$$M = \exp(\alpha B) \sum_{n=0}^{\infty} m_n, \quad m_n \sim \alpha^n, \quad (13)$$

and B can be obtained just from the lowest order diagram. Introducing an IR cut-off λ , $B \sim -\log m^2/\lambda^2$, which indeed shows that when we remove the cut-off the probability of finding an isolated charged fermion is zero in QED. The addition of soft photons changes that result, multiplying the total amplitude by a factor $\sim (\Delta E/\lambda)^2$. There is a trade between the infrared regulator and $\Delta E, \Delta\theta$. The latter are, of course, detector-dependent quantities.

Although only partial results exist[8], it is believed that a similar exponentiation takes place in QCD. Due to the confinement subtleties it is unclear whether the suppression factor is compensated by radiation of soft gluons. Even if this compensation does actually take place that would not disprove confinement, only that confinement would have nothing to do with the structure of infrared singularities of the theory.

The previous discussion can be summarized in the following way: due to IR singularities one is forced to consider cross sections not of individual particles in the final state, but rather of bunches of particles, each ‘hard’ quark and gluon surrounded by a ‘soft’ cloud of gluons and, perhaps, quarks. These bunches are called ‘jets’.

The Bloch-Nordsieck and Kinoshita-Lee-Nauenberg theorems guarantee the finiteness of the cross-sections. We have to define an energy and angle resolution. For instance, if p is the momentum of a primary quark we can impose that the energy of each soft particle in its jet satisfies $k_i^0 < \epsilon p_0$ and also that $\arg(\vec{p}, \vec{k}_i) < \delta$. We will get singularities of the form $\alpha_s \log \epsilon \log \delta$ when $\epsilon, \delta \rightarrow 0$. The specific details depend on the precise definition of the jet.

3 Free Parton Model

The counterpart of having an effective coupling constant which grows at low energies is that the theory becomes simple at high energies, making perturbative calculations possible (at least *some* of them). Indeed a brilliant confirmation of the existence of nearly free constituents inside the nucleon was provided more than twenty years ago by a series of experiments carried out at SLAC[9]. Then it became possible to scatter electrons off nucleons in fixed target experiments with a typical momentum transfer $\sim 1 - 10 \text{ (GeV)}^2$, a kinematical range unexplored until that time. The kinematics of Deep Inelastic Scattering (DIS) processes is shown in fig. 4.

The virtual intermediate boson is far off its mass-shell and scatters off a quark or gluon in a time of $\mathcal{O}(1/\sqrt{-q^2})$. Typically quarks and gluons are themselves off-shell by an amount of $\mathcal{O}(\Lambda_{QCD})$. After the scattering the outgoing particles recombine into hadrons in a time of $\mathcal{O}(1/\Lambda_{QCD})$. Thus Deep Inelastic is a two-step process

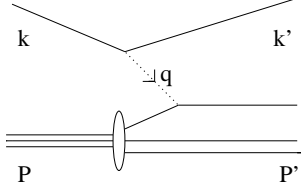


Figure 4: The kinematics of deep inelastic scattering.

- Short distance scattering occurs with a large momentum transfer. Well described by perturbation theory.
- Outgoing particles recombine. Not calculable in perturbation theory.

However, the second part can be side-stepped all together for fully inclusive rates. Then perturbation theory is adequate to describe many features of DIS.

If we place ourselves in the center of mass of the hadron and virtual intermediate boson both particles move very fast towards each other. Whatever components the hadron contains they will all have moments parallel to P^μ , up to transversal motion of $\mathcal{O}(\Lambda_{QCD})$. Let us write

$$p^\mu = xP^\mu. \quad (14)$$

The squared CM energy of the lepton and proton constituent will be

$$\hat{s} = (xP + k)^2 \simeq 2xPk \simeq xs. \quad (15)$$

We neglect masses (as well as the fact that constituents are off-shell by $\mathcal{O}(\Lambda_{QCD})$) The final momentum of the constituent is $xP + q$. Therefore

$$0 \simeq (xP + q)^2 \simeq 2xPq + q^2, \quad (16)$$

so $x = -q^2/2Pq$. If ν is the energy transfer in the LAB system, we can also write

$$x = \frac{-q^2}{2\nu m_N} \quad (17)$$

m_N being the nucleon mass. It is convenient to introduce

$$y = \frac{Pq}{Pk} = 1 - \frac{Pk'}{Pk} \quad (18)$$

In the lab frame $y = \nu/E$ and $0 \leq y \leq 1$. y is thus the relative energy loss of the colliding lepton.

Let us for the time being ignore altogether QCD interactions and let us assume that constituents of the nucleons (which we will call partons) are free. DIS will then be described by an incoherent sum over elementary processes. The partonic differential cross sections in the LAB frame will be

- $\nu q, \bar{\nu}q$ -scattering

$$\frac{d\hat{\sigma}_\nu}{dy} = \left(\frac{g^2}{4\pi}\right)^2 \frac{\pi m E}{(q^2 - M_W^2)^2} [g_L^2 + g_R^2(1-y)^2], \quad (19)$$

$$\frac{d\hat{\sigma}_{\bar{\nu}}}{dy} = \left(\frac{g^2}{4\pi}\right)^2 \frac{\pi m E}{(q^2 - M_W^2)^2} [g_R^2 + g_L^2(1-y)^2]. \quad (20)$$

- $e q$ -scattering

$$\frac{d\hat{\sigma}_e}{dy} = Q^2 \frac{4\pi\alpha^2 m E}{q^4} [1 + (1-y)^2]. \quad (21)$$

The neutral current sector is dominated by γ exchange below $q^2 = M_Z^2$, so we have not bothered to include Z exchange. In (21) Q is the quark electric charge (in units of e) and m is the target mass. Since $p^\mu = xP^\mu$, we just take $m = xm_N$. Then, for instance,

$$\frac{d^2\hat{\sigma}_e}{dxdy} = Q^2 \frac{4\pi\alpha^2 xm_N E}{q^4} [1 + (1-y)^2]. \quad (22)$$

Let $u(x)dx, d(x)dx, \dots$ be the number of u, d, \dots quarks with momentum fraction between x and $x+dx$ in a nucleon. Then $xu(x), xd(x), \dots$ will be the fraction of the nucleon momentum carried by u, d, \dots quarks. We, of course, identify quarks with partons and, since we assume that they are free, proceed to sum incoherently over the different scattering possibilities. For instance in $ep \rightarrow eX$

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{s} \frac{1 + (1-y)^2}{xy^2} \left[\frac{4}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(d(x) + \bar{d}(x)) + \frac{1}{9}(s(x) + \bar{s}(x)) \right]. \quad (23)$$

(We neglect here the possible contribution from the sea of heavy quarks in the nucleon.) Other DIS processes weigh differently quarks and antiquarks. For instance, in $\nu p \rightarrow \mu X$ if $-q^2 \ll M_W^2$ we have

$$\frac{d^2\sigma}{dxdy} = x \frac{G_F^2 s}{\pi} [c_c^2 d(x) + s_c^2 s(x) + \bar{u}(x)(1-y)^2], \quad (24)$$

with $c_c = \cos \theta_c$, $s_c = \sin \theta_c$, the cosinus and sinus of the Cabibbo angle, respectively.

The parton distribution functions (PDF) $q(x)$ are quantities which are not calculable within perturbative QCD, as we will see.

Probably the first thing that one learns is that gluons are very important. From the SLAC-MIT data[9]

$$Q = U + D + S = \int_0^1 dx x(u(x) + d(x) + s(x)) \simeq 0.44, \quad (25)$$

$$\bar{Q} = \bar{U} + \bar{D} + \bar{S} = \int_0^1 dx x(\bar{u}(x) + \bar{d}(x) + \bar{s}(x)) \simeq 0.07. \quad (26)$$

The total fraction of momentum carried by quarks (and antiquarks) is only about 50%. The rest is carried by gluons (parametrized by a PDF $g(x)$), showing that although the naive quark model works very well is just a gross simplification as a model of hadrons, at least at large $-q^2$. In fact we know the asymptotic values of (25) and (26) based in the equipartition of energy in a free theory (since, asymptotically, QCD is free)

$$\int_0^1 dx x q(x) \rightarrow \frac{3N_f}{16 + 3N_f} \quad \int_0^1 dx x g(x) \rightarrow \frac{16}{16 + 3N_f}. \quad (27)$$

From the above limiting values we see and at higher energies the total momentum carried by *constituent* or *valence* quarks diminishes and that an equally important role is played by particles from the Dirac sea of the nucleon.

Another example where the quark model fails to describe some basic features of hadrons is provided by the ‘spin of the proton’ problem[4]. μ -scattering on polarized targets shows that the fraction of the total spin of the proton that can naively be associated to constituent quarks is surprisingly small. We shall not dwell on this matter further here.

Nevertheless, there are some obvious sum rules for the parton distribution functions which can ultimately be explained in terms of the quark model. For the proton

$$\int_0^1 dx (u(x) - \bar{u}(x)) = 2, \quad (28)$$

$$\int_0^1 dx (d(x) - \bar{d}(x)) = 1, \quad (29)$$

$$\int_0^1 dx (s(x) - \bar{s}(x)) = 0. \quad (30)$$

On QCD grounds we expect that this free parton model description of the hadrons becomes more and more accurate when $-q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, while keeping x fixed. This limit is known as Bjorken scaling and in the strict $-q^2 = \infty$ limit everything depends just on x .

Let us now try to rederive the previous results in a more theoretical setting. Let us consider for instance νp scattering. Then

$$\frac{d^2\sigma}{d(-q^2)d\nu} = \frac{G_F^2 m_N}{\pi s^2} L^{\mu\nu} H_{\mu\nu}, \quad (31)$$

where

$$L^{\mu\nu} = \frac{1}{8} \text{Tr}[\gamma^\mu(1 - \gamma_5)\gamma^\alpha\gamma^\nu(1 - \gamma_5)\gamma^\beta]k_\alpha k_\beta \quad (32)$$

is the trace over the leptonic external lines, and $H_{\mu\nu}$ is given by

$$\sum_X \langle P | J_\mu(0) | X(P') \rangle \langle X(P') | J_\nu(0) | P \rangle = \int d^4 z e^{iqz} \langle P | J_\mu(z) J_\nu(0) | P \rangle, \quad (33)$$

which is just $\text{Im}\Pi_{\mu\nu}(q)$, with

$$\Pi_{\mu\nu}(q) = \int d^4 z e^{iqz} \langle P | T J_\mu(z) J_\nu(0) | P \rangle. \quad (34)$$

We decompose $H_{\mu\nu}$ as

$$H_{\mu\nu} = -g_{\mu\nu}F_1 + \frac{P_\mu P_\nu}{\nu m_N}F_2 + \frac{i}{2\nu m_N}\epsilon_{\mu\nu\rho\sigma}P^\rho q^\sigma F_3 \quad (35)$$

(If we assume that we are working with non-polarized targets P and q are the only vectors at our disposal.) F_1 , F_2 and F_3 are called the nucleon structure functions. Using the kinematical relations $x = -q^2/2\nu m_N$ and $y = 2m_N\nu/s$ we get

$$d(-q^2)d\nu = \nu s dx dy, \quad (36)$$

$$\frac{d^2\sigma}{dx dy} = \frac{G_F^2 s}{2\pi} [F_1 xy^2 + F_2(1-y) - F_3 xy(1 - \frac{y}{2})]. \quad (37)$$

Let us now compare with the free parton model. We see that (restoring the νp index, to make apparent that the structure functions are process dependent)

$$F_1^{\nu p}(x) = c_c^2(\bar{u}(x) + d(x)) + s_c^2(s(x) + \bar{u}(x)), \quad (38)$$

$$F_2^{\nu p}(x) = 2xc_c^2(\bar{u}(x) + d(x)) + 2xs_c^2(s(x) + \bar{u}(x)), \quad (39)$$

$$F_3^{\nu p}(x) = 2c_c^2(\bar{u}(x) - d(x)) + 2s_c^2(-s(x) + \bar{u}(x)). \quad (40)$$

For other processes the actual expressions may vary but the structure functions are always linear combinations of the parton distribution functions, i.e. $F_2(x) = x \sum_f \delta_f q_f(x)$, etc.

Note that in the free parton model

$$F_L(x) = F_2(x) - \frac{F_1(x)}{2x} = 0. \quad (41)$$

This is the Callan-Gross relation, which actually is not an exact one; it gets modified when the q^2 dependence is included, i.e. we depart from the strict $-q^2 = \infty$ limit.

An exact sum rule, which is easily expressed in terms of the structure function $F_2(x)$ was given by Adler

$$\int_0^1 \frac{dx}{x} (F_2^{\nu I} - F_2^{\nu \bar{I}}) = 4\langle I_3 \rangle \quad (42)$$

where I_3 is the third component of the target of isospin I . Other sum rules are not exact, except in the strict free parton model, but their violations are computable within perturbative QCD. Two instances are the Gross-Llewellyn-Smith sum rule

$$\frac{1}{2} \int_0^1 dx (F_3^{\nu p} + F_3^{\nu \bar{p}}) = \int_0^1 dx (u(x) - \bar{u}(x) + d(x) - \bar{d}(x)) + \mathcal{O}(\alpha_s) = 3 + \mathcal{O}(\alpha_s), \quad (43)$$

and the Gottfried sum rule

$$\int_0^1 \frac{dx}{x} (F_2^{\mu p} - F_2^{\mu n}) = \frac{1}{3} + \mathcal{O}(\alpha_s) + \int_0^1 dx (\bar{u}(x) - \bar{d}(x)). \quad (44)$$

Violations to the different sum rules are a theoretically clean way to extract α_s . Of course in practice things are difficult because the sum rules involve an integral over all values of x , which is always poorly known in some range of the integrand and extrapolations are needed.

If we assume that α_s is extracted from some other source, the Gottfried sum rule provides some interesting information on the sea contents of light antiquarks in the nucleon. The collaboration NMC[10] has determined that for the proton

$$\bar{U} - \bar{D} \equiv \int_0^1 dx (\bar{u}(x) - \bar{d}(x)) = -0.15 \pm 0.036. \quad (45)$$

In the proton quark sea there are many more d -type antiquarks than u -type. This is in fact a large isospin violation, much larger than expected to the mass difference of the quarks and, in fact, goes, at least naively, in the opposite direction.

The isospin violation is larger at low values of x ($x \leq 0.2$) and also at low values of Q^2 , which hints that long-distance physics (quite remote from perturbative QCD) is called for. The enhancement of d -type antiquarks is confirmed by other experiments. For instance Na51[11] finds at $x = 0.18$ $\bar{u}/\bar{d} = 0.51 \pm 0.09$, while NuSeaC[12] finds at $Q^2 = 7.4 \text{ GeV}^2$ that $\bar{U} - \bar{D} = -0.10 \pm 0.024$.

Could this be due to the exclusion principle, as illustrated in figure 5, which makes harder for u -type antiquarks to appear in the sea? It could be, but it is very difficult to come up with quantitative results. A partial understanding is provided by the chiral quark model, including pion exchange (see figure 5), but this type of physics is still very poorly understood.

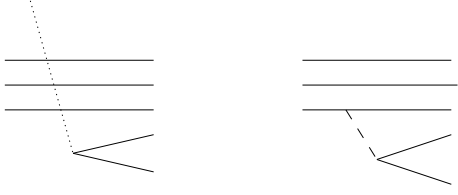


Figure 5: Valence and sea quarks. Contribution from the chiral quark model.

4 Scaling Violations

It is plain clear from the data that there is some $Q^2 = -q^2$ dependence in the structure functions. In other words, there are violations of Bjorken scaling and actually $F_f = F_f(x, Q^2)$. The free parton model is not completely correct (no big surprise, of course). Our job is to try to understand these violations in the framework of QCD.

Let us assume that we have isolated a parton with initial momentum $p = xP$. The probability of finding such a parton is given by $q(x)$. At the parton level the structure function F_2 is just $\hat{F}_2 = x\delta_f$ (δ_f is the appropriate charge). At the proton level, however, this partonic cross-section has to be multiplied by the probability of finding the parton with momentum fraction x , i.e. by $q(x)$. We write this in the form

$$F_2(x) = x\delta_f \int_0^1 d\xi q(\xi) \delta(x - \xi) = x\delta_f \int_0^1 \frac{d\xi}{\xi} q(\xi) \delta(1 - \frac{x}{\xi}). \quad (46)$$

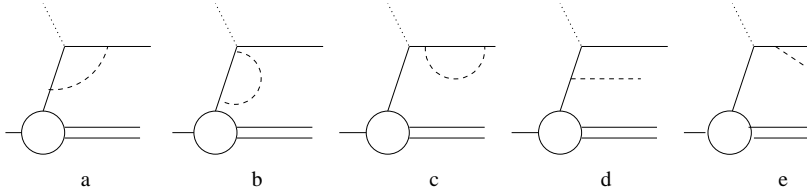


Figure 6: Contribution at $\mathcal{O}(\alpha_s)$ to the relevant DIS subprocess.

At $\mathcal{O}(\alpha_s)$ many diagrams contribute. They are given in figure 6. We are looking for scaling violations and, therefore, we must look for logs. In other words we must investigate ultraviolet, infrared and mass singularities of any kind. It turns out that ultraviolet singularities are proportional to the free result and simply renormalize δ_f . Infrared divergences cancel amongst all diagrams. Only the mass singularity present in diagram (d) when the momentum of the gluon is parallel to that of the gluon survives. Keeping only logarithmic terms, the calculation at $\mathcal{O}(\alpha_s)$ amounts to the replacement

$$\delta(1 - \frac{x}{\xi}) \rightarrow \delta(1 - \frac{x}{\xi}) + \frac{\alpha_s}{2\pi} P(\frac{x}{\xi}) \log \frac{Q^2}{\lambda^2}, \quad (47)$$

where λ^2 is an infrared regulator and

$$P(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right]. \quad (48)$$

Only the logarithmic term is retained for this discussion. Now we understand the reason for writing things in apparently such a complicated way. First of all, scaling violations appear through the contribution of real soft particles. ξ is the original momentum fraction of the nucleon carried by the parton, which is reduced to $x \leq \xi$ after the emission of the soft gluon. The hard scattering takes place with the parton carrying fraction x . All along the discussion, the transverse motion of the partons inside the target is neglected as well as are all masses.

Then

$$F_2(x) = x \delta_f [q_0(x) + \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \frac{\alpha_s}{2\pi} P\left(\frac{x}{\xi}\right) \log \frac{Q^2}{\lambda^2}]. \quad (49)$$

In addition we have replaced $q(x)$ by $q_0(x)$, the bare PDF. If we now define the renormalized PDF by

$$q(x, \mu^2) = q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_0(\xi) P\left(\frac{x}{\xi}\right) \log \frac{Q^2}{\lambda^2}, \quad (50)$$

we can write

$$F_2(x, Q^2) = x \delta_f [q(x, \mu^2) + \int_x^1 \frac{d\xi}{\xi} q(x, \mu^2) \frac{\alpha_s}{2\pi} P\left(\frac{x}{\xi}\right) \log \frac{Q^2}{\mu^2}]. \quad (51)$$

No doubt the similarity with the usual renormalization process did not go unnoticed. Now the infrared regulator has been eliminated (hidden in the bare PDF) at the expense of introducing a renormalization-scale dependence. These are the sought after scaling violations.

5 Altarelli-Parisi Equations and Λ_{QCD}

At this point it is convenient to introduce the variable $t = \frac{1}{2} \log \mu^2 / \Lambda_{QCD}^2$. It then follows from (50) that

$$\frac{\partial}{\partial t} q(x, t) = \frac{\alpha_s(t)}{\pi} \int_x^1 \frac{d\xi}{\xi} q(\xi, t) P\left(\frac{x}{\xi}\right), \quad (52)$$

which immediately translate into differential equations for the structure functions themselves. These are the Altarelli-Parisi equations[13]. They summarize the rate of change of the parton distribution functions with t .

We define the moments of the PDF's by

$$q(n, t) = \int_0^1 dx x^{n-1} q(x, t). \quad (53)$$

Introducing the anomalous dimension γ_n as

$$\gamma_n = \int_0^1 dx x^{n-1} P(x), \quad (54)$$

the convolution over the fractional momentum ξ transforms into a product

$$\frac{\partial}{\partial t} q(n, t) = \frac{\alpha_s(t)}{\pi} \gamma_n q(n, t). \quad (55)$$

This leads to following scaling behaviour for the moments of the structure functions

$$F_2(n, Q^2) = F_2(n, Q_0^2) \left(\frac{\alpha_s(Q_0)}{\alpha_s(Q)} \right)^{\frac{\gamma_n}{\beta_1}}, \quad (56)$$

which is our final expression. Experiments agree on the whole very nicely with the scaling violations predicted by QCD. Taking into account all the subtle points of Quantum Field Theory that have gone into the analysis, this provides a beautiful check of the theoretical framework.

We have been considering F_2 , but the same procedure can be repeated for any structure function. The expression (56) amounts to resumming the leading logs obtained by iteration of soft collinear gluons. The diagram is the one shown in figure 7.

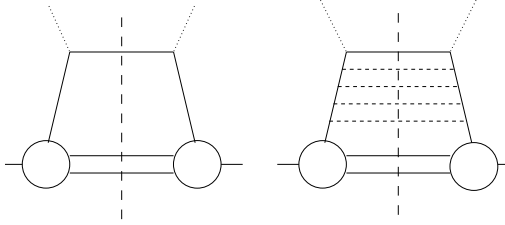


Figure 7: Handbag and ladder diagrams.

As a simplifying hypothesis we have neglected mixing. In fact, the evolution equation is a $(2N_f + 1) \times (2N_f + 1)$ matrix, involving quarks and gluons. In the flavour singlet case life is more complicated; there is mixing with gluon operators and therefore one must also consider gluon parton distribution functions as well

$$\frac{\partial q(x, t)}{\partial t} = \frac{\alpha_s(t)}{\pi} \int_x^1 \frac{dy}{y} [q(y, t) P_{qq}\left(\frac{x}{y}\right) + g(y, t) P_{gq}\left(\frac{x}{y}\right)] \quad (57)$$

$$\frac{\partial g(x, t)}{\partial t} = \frac{\alpha_s(t)}{\pi} \int_x^1 \frac{dy}{y} [g(y, t) P_{gg}\left(\frac{x}{y}\right) + q(y, t) P_{qg}\left(\frac{x}{y}\right)] \quad (58)$$

The detailed form of the Altarelli-Parisi kernels at leading and NLO order can be found in [14]. No complete calculation exists yet at the NNLO to my knowledge, just some partial results.

It is important to realize that the Altarelli-Parisi equations are not exact. They take into account the perturbative contribution only (and this up to a given order in perturbation

theory). They also neglect transverse motion, which leads to corrections of $\mathcal{O}(\Lambda_{QCD}/Q^2)$ to the leading results. These are more easily dealt with in the perhaps more rigorous (but more cumbersome) treatment based in the Operator Product Expansion[15], which will not be discussed here. Target mass corrections should be equally taken into account. They are particularly important near thresholds (such as the charm and bottom thresholds). For instance the proper treatment of thresholds is highly relevant for HERA, since a big chunk of the data comes from a region close to these thresholds. And, of course, mass corrections are important for charm PDF from the sea, which have actually been recently measured.

The analysis of Deep Inelastic Scattering based on the Altarelli-Parisi equations (or, alternatively, on the Operator Product Expansion) has been one of the most clear tests of perturbative QCD and traditionally the best way of determining α_s , which, as we have seen, enters in the scaling violations. However, at present the value of $\alpha_s(M_Z)$ extracted from Z-physics is equally accurate if not more.

For some time a discrepancy was claimed between the value of α_s obtained from the analysis of scaling violations and the Z-pole value. For instance the value quoted by CCFR was $\alpha_s(M_Z) = 0.111 \pm 0.004$, well below the world average[2], and quite away, for instance, from measurements based on event shapes at the Z peak ($\alpha_s(M_Z) = 0.122 \pm 0.007$; for measurements of α_s at LEP, see e.g.[17]). The average α_s from DIS was given[18] just two years ago to be $\alpha_s(M_Z) = 0.113 \pm 0.005$. Theoretical speculations were fuelled and it was claimed that non-perturbative corrections could be larger than originally thought.

Deep inelastic scattering data have been reanalyzed recently and the quoted value for $\alpha_s(M_Z)$ from DIS from a global fit[19] to all data is $\alpha_s(M_Z) = 0.118 \pm 0.005$. The agreement with other determinations is now almost perfect. The discrepancy was apparently due, it is claimed, to the energy calibration of the detector (in the case of CCFR, at least; the new CCFR value is $\alpha_s(M_Z) = 0.119 \pm 0.005$ [16]), a better understanding of higher twist corrections (not discussed here) and from the treatment of the contribution from heavy quarks from the sea, charm in particular. We will see later, when we discuss in somewhat more detail the form of the PDF, that one must actually make a number of hypothesis before being able to extract $\alpha_s(M_Z)$. While do not regard the issue as totally settled yet, because some of the older data sets are very poorly described by the now preferred value of $\alpha_s(M_Z)$, the most recent data coming from HERA[20] (H1 and ZEUS) give values which agree nicely with each other and in fact fall well in the high α_s range, almost on top of the new average value. These ongoing experiments will become statistically more and more significant in the near future in the determination of $\alpha_s(M_Z)$ as more and more data points pile up.

6 Parton Distribution Functions

We do not know in general how to compute the parton distribution functions, even for $-q^2 \rightarrow \infty$. Only their evolution can be reliably computed either through the Operator Product Expansion or the use of the Altarelli-Parisi equations and this for large enough values of $-q^2$. The scaling behaviour is governed by the anomalous dimensions. At leading order they are

$$\gamma_{qq}(j) = C_F \left[-\frac{1}{2} + \frac{1}{j(j+1)} - 2 \sum_{k=2}^j \frac{1}{k} \right], \quad (59)$$

$$\gamma_{qg}(j) = T_R \left[\frac{2+j+j^2}{j(j+1)(j+2)} \right], \quad (60)$$

$$\gamma_{gq}(j) = C_F \left[\frac{2+j+j^2}{j(j^2-1)} \right], \quad (61)$$

$$\gamma_{gg}(j) = 2C_A \left[-\frac{1}{12} + \frac{1}{j-1} + \frac{1}{(j+1)(j+2)} - \sum_{k=2}^j \frac{1}{k} \right] - \frac{2N_f}{3} T_R, \quad (62)$$

where C_F, T_R and C_A are group-theoretical factors.

An interesting issue is the behaviour of the parton distribution functions at the endpoints $x = 0$ and $x = 1$. The large n behaviour of the moments probes the $x \rightarrow 1$ region. Since it is natural to expect that at the kinematical boundaries the parton distribution functions vanish, one can make the following ansatz for $x \rightarrow 1$

$$q(x, Q^2) \sim A(Q^2)(1-x)^{\nu(\alpha_s(Q^2))-1}. \quad (63)$$

Demanding that eq. (63) fulfills the q^2 evolution equation leads to

$$A(Q^2) = A_0 \frac{[\alpha_s(Q^2)]^{-d_0}}{\Gamma(1 + \nu(\alpha_s(Q^2)))} \quad \nu(\alpha_s) = \nu_0 - \frac{16}{33 - 2N_f} \log \alpha_s(Q^2), \quad (64)$$

$$d_0 = \frac{16}{33 - 2N_f} \left(\frac{3}{4} - \gamma_E \right). \quad (65)$$

Likewise, for the gluons we have

$$g(x, Q^2) \sim A'_0 \frac{[\alpha_s(Q^2)]^{-d_0}}{\Gamma(2 + \nu(\alpha_s(Q^2)))} \frac{(1-x)^{\nu(\alpha_s(Q^2))}}{\log(1-x)}. \quad (66)$$

The constants A_0, A'_0 and ν_0 are not calculable on perturbative QCD and depend on the specific operator. d_0 is universal.

When $x \rightarrow 1$ the gluon distribution functions approach zero more rapidly than the quark ones. For large values of x the quark contents of nucleons is the relevant one. Second order

corrections to this asymptotic behaviour can be derived in a similar way and are known. It turns out that the correction is arbitrarily large if one gets sufficiently close to $x = 1$. This is because the collinear gluon is, in addition, soft in that exceptional configuration thus giving rise to a $\log(1 - x)$ singularity. Multiple emission is then kinematically favoured, since the log overcomes the α_s suppression.

For small values of x the opposite behaviour takes place, the gluon distribution function eventually becomes dominant. At LHC the cross-section will be greatly dominated by low- x physics and the important process there will be gluon-gluon scattering. At the current Tevatron run the quark contents of protons and antiprotons is still dominant. Let us see why gluons dominate completely at low x .

The key point is the appearance of the singularity for $j = 1$. Indeed, as $j \rightarrow 1$

$$\gamma_{gg}(j) \sim \frac{2N}{j-1}. \quad (67)$$

Then

$$g(j, t) = g(j, t_0) \exp\left[\frac{N}{\pi\beta_1(j-1)} \log \frac{t}{t_0}\right]. \quad (68)$$

Then we proceed to evaluate $g(x, t)$ by performing an inverse Mellin transform

$$xg(x, t) = \frac{1}{2\pi i} \int_C dx^{1-j} g(j, t). \quad (69)$$

The integration circuit is a line in the direction of the imaginary axis in the complex j plane, to the right of the $j = 1$ singularity. The saddle point method can now be used provided that $\log(1/x)$ is large and this is the reason why this procedure gives only the small x behaviour. Working things out we see that the gluon parton distribution function for low x behaves as

$$g(x) \sim \frac{1}{x} \exp \sqrt{C(Q^2) \log \frac{1}{x}}, \quad (70)$$

where $C(Q^2)$ is calculable. Unfortunately, this answer is not totally satisfactory because something must stop the growth in $g(x)$ for low x , or else one runs into unitarity problems sooner or later, and thus eq. (70) it is not credible all the way to $x = 0$. Technically speaking, there must be corrections that destabilize the saddle point solution. Physically, the uncontrolled growth of the gluon distribution is an infrared instability. The density of soft gluons is too large. Shadowing and non-linear evolution equations are the buzzwords here[21].

The double scaling limit (high Q^2 , low x) is well supported by the data. See for instance[22] for a recent analysis coming from measurements of F_2 at H1.

Except in these two limiting cases, PDF's have to be parametrized. The way one proceeds is by proposing a given parametrization at some reference value Q_0^2 , then evolve to all desired values of Q^2 using the Altarelli-Parisi equations, then perform a global fit of the parameters describing the PDF and, at the same time, determine α_s .

7 Confinement

Λ_{QCD} sets a natural scale in the theory. Well above Λ_{QCD} perturbation theory makes sense. Of course perturbative QCD at large enough energies describes a world of quasi-free quarks, interacting with Coulomb-like forces. We know very well that hadronic physics is a very different world where quarks are confined into colorless hadrons. As soon as $q^2 \sim \Lambda_{QCD}^2$ perturbation theory is unreliable. It simply cannot explain confinement.

What confinement means is that there is a force between quarks that does not decrease with distance. There is indeed phenomenological evidence (which is supported by lattice analysis) that the interquark potential at large distances in QCD is of the form

$$V(r) \sim a\Lambda_{QCD}^2 r - \frac{b}{r} + \dots \quad (71)$$

The first term is a confining quark potential. The constant a has to be ~ 1 because Λ_{QCD}^2 is the only dimensional quantity at our disposal. The Coulombic part is called the Lüscher term and plays a crucial role in heavy quark spectroscopy[23].

At short distances the behaviour of the interquark potential is totally different. It is Coulomb-like

$$V(r) \sim -\frac{\alpha_s}{r}. \quad (72)$$

Different quarks probe the different regimes. Indeed, because $\alpha_s(m_t)$ is so small (say ~ 0.1), the corresponding Bohr radius is $r_0 \sim 10^{-2}$ fm, much smaller than Λ_{QCD}^{-1} . The coulombic part of the interquark potential largely dominates. (At such short distances the linearly rising potential is not at work, the leading confinement effects are $\sim r^3$, as discussed by Leutwyler some time ago[24], but they can be safely neglected at first approximation.)

Bottom and charm are in a somewhat intermediate position. $\alpha_s(m_b)$ is still relatively small. The Bohr radius is 10^{-1} fm, smaller but comparable to Λ_{QCD}^{-1} . Spectroscopy is basically perturbative, at least for the lowest levels, but some non-perturbative effects are visible. Charm is really no-man's land. Both perturbative and non-perturbative effects compete even for the ground state $n = 1$. For light quarks the Bohr radius is several fm and the confining potential is fully at work.

The existence of a confining potential leads to very large multiplicities and jets. One can imagine a quark-antiquark being formed at the primary vertex then moving apart. Part of their kinetic energy is deposited in the interquark potential as they move away. Very quickly a separation r_m is reached where the energy deposited is enough to form a new quark-antiquark pair,

$$\Lambda_{QCD}^2 r_m \simeq 2m_q, \quad (73)$$

at that moment the quark-antiquark ‘string’ breaks and the process is repeated until the average relative momentum is small enough and hadronization takes place.

There is a lot of physics in the string picture. We can think of color forces being confined in some sort of tube or string joining the two moving quarks. The chromodynamic energy is thus stored in a relatively small region of space-time. If this picture is correct we should expect hadronization to take place in this region in preference to any other. This is indeed the case; in three jet events (which originate from $\bar{q}qg$, with a hard gluon) there is a clear enhancement of soft gluon and hadron production in the regions between color lines (representing the gluon by a double color line, or $\bar{q}q$ state), and a relative depletion in other regions. This phenomenon is called color coherence[25].

8 Dual Models

We now backtrack in history, to the pre-QCD days, and recall that in the 60’s the duality hypothesis was much in fashion. The hypothesis stated that in strong interactions a sum over intermediate states in the s -channel should reproduce the sum over resonances in the t -channel. Mathematically,

$$A(s, t) = \sum_J \frac{g_J^2 s^J}{t - M_J^2} = \sum_J \frac{g_J^2 t^J}{s - M_J^2}. \quad (74)$$

Of course for this to have even a chance of being true an infinite number of intermediate states is required. It should be stated right away that the evidence for this peculiar property was (and still is) rather weak.

However, in 1968, Veneziano[26] took the idea seriously and proposed the following amplitude

$$A(s, t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}, \quad \alpha(s) = \alpha(0) + \alpha' s. \quad (75)$$

This amplitude is manifestly dual. In 1969 and 1970 Y.Nambu and others unveiled the relation between the Veneziano amplitude and open string theory. It was later generalized to closed strings by Koba and Nielsen.

This of course is the way to make contact with the long-distance properties of QCD that we have discussed in the previous section. If in some kinematical regime QCD can be described by some type of string theory, an amplitude of the Veneziano type should describe strong interactions in a regime where perturbation theory is not valid.

Unfortunately life is not so easy. First of all, consistency of string theory requires $\alpha(0) = 1$ and then the Veneziano amplitude exhibits poles in the s -channel whenever $s = (n - 1)/\alpha'$, $n = 0, 1, \dots$. There is a tachyonic scalar/pseudoscalar particle and a massless vector particle. In addition, the amplitude does not exhibit the proper chiral behaviour (Adler zero), i.e. $A(0, 0) = 0$, if we are to interpret the pseudoscalars as pions. It is a complete phenomenological failure.

Lovelace and Shapiro[27] finally proposed an amplitude with the correct behaviour or, at least, not manifestly incorrect. It is inspired from the theory of supersymmetric strings and the equivalent of the Veneziano amplitude now reads

$$A(s, t) = \frac{\Gamma(1 - \alpha(s))\Gamma(1 - \alpha(t))}{\Gamma(1 - \alpha(s) - \alpha(t))}, \quad \alpha(s) = \alpha(0) + \alpha's, \quad (76)$$

where $\alpha(0)$ in principle also equals 1 here. Again this amplitude (which is tachyon-free — this is the main virtue of the supersymmetric string) is not physically acceptable because the amplitude does not have the correct Adler zero. However, it does reproduce the right chiral behaviour if one replaces by hand this value by $1/2$ and is then appropriate to describe pion scattering. Unfortunately this replacement cannot be derived at present from any known string theory. The corresponding trajectory is called the Regge trajectory and corresponds to the exchange of open strings and, as discussed, has an intercept $\alpha_R = 1/2$. Physically this is interpreted as the exchange of quark-antiquark pairs. In addition there is a Pomeron trajectory which has an intercept $\alpha_P = \alpha_R + 1/2$, and is due to the exchange of closed strings (interpreted as glueballs in QCD).

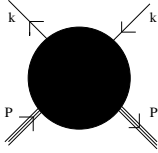


Figure 8: Nucleon-parton scattering process.

Let us now return to DIS and let us see what all this has to do with it. Let us consider the behaviour of the amplitude $A(s, t)$ for large s and fixed t

$$A(s, t) \sim s^{\alpha't + \alpha_0}. \quad (77)$$

Let us now assume that the elastic nucleon-parton amplitude (figure 9) is described by such an amplitude. Kinematically, $s = (P - k)^2$. If we decompose

$$k = xP - \frac{k_T^2}{2x}n + k_T, \quad (78)$$

where n is a vector such that $n^2 = 0$, $n \cdot P = 1$ and $k_T \cdot n = k_T \cdot P = 0$, then, with the usual approximations,

$$s = -2k \cdot P = -\frac{k_T^2}{x} \quad (79)$$

We see then that, at a finite value of k_T , low x corresponds to the large s behaviour. On the other hand, for this subprocess $t = 0$, and we of course realize that the amplitude is directly related to the cross-section for parton + proton \rightarrow anything. If the parton is a quark only the Reggeon will contribute. If there is mixing with partonic gluons we will have a contribution from both the Regge and Pomeron trajectories. After a short calculation we shall conclude that

$$F_2(x) \sim A_P x^0 + A_R x^{\frac{1}{2}}. \quad (80)$$

And, therefore that, at low values of x ,

$$g(x) \sim x^{-1}, \quad q(x) \sim x^{-\frac{1}{2}}. \quad (81)$$

These are the implications of Regge theory for the parton distribution functions. Notice that there is no Q^2 dependence anywhere, so the question poses itself as to which is the appropriate value of Q^2 to compare with. The answer is not very well defined, but it should correspond to the typical range of energies where Regge phenomenology is known to be valid in other contexts, i.e. a few GeV. In fact, a fit to the gluon PDF shows that at $Q^2 = 4 \text{ GeV}^2$, $g(x) \sim x^{-1.17}$. Not bad.

9 Low x region

The region below 10^{-2} had not been explored experimentally until very recently; a first look at these low- x values has been provided by the commissioning of HERA. HERA is a machine ideally suited for an in-depth analysis of structure functions. It should be possible to arrive at very low values of x (down to $x \sim 10^{-5}$).

Most parametrizations have traditionally performed very poorly when extrapolated to the low x region. Typically they predict an increase as $x \rightarrow 0$ which is lower than what is actually seen. The behaviour $F_2(x) \sim x^{-\lambda}$, with $\lambda \sim 1/2$ as $x \rightarrow 0$, which is predicted from the BFKL evolution equation[28] seemed at some point (see e.g.[29]) to stand the comparison

with HERA results best. However, this behaviour is still incompatible with unitarity and cannot hold all the way to $x = 0$ either. In fact we know now that the predictions from BFKL cannot be trusted. This has prompted a renewed interest in trying to extract the behaviour at low x from conventional Altarelli-Parisi evolution. The consensus now seems to be that even for the low values of x analyzed at HERA there is no real evidence of any results beyond ordinary perturbative QCD.

It is easy to understand why perturbative QCD must fail at some point. The expansion of the splitting function $P(z)$ in powers of α_s at the NLO actually resums all terms of the form $(\alpha_s \log Q^2)^n$ and $\alpha_s^n \log^{n-1} Q^2$. Looking at (10) we see that the propagator causing the mass singularity is ($p = \xi P$)

$$\frac{1}{2pk} = -\frac{2x}{\xi k_T^2}. \quad (82)$$

Apart from the parametric integrals, we have

$$\int \frac{d^2 k_T}{k_T^2}. \quad (83)$$

This is the origin of the $\log \lambda^2$ and, eventually, of the $\log Q^2$.

The leading $\log^n Q^2$ will thus be produced by one single region in integration

$$\int \frac{d^2 k_T^n}{(k_T^n)^2} \int \frac{d^2 k_T^{n-1}}{(k_T^{n-1})^2} \cdots \int \frac{d^2 k_T^1}{(k_T^1)^2}, \quad (84)$$

with $|Q| \gg |k_T^n| \gg |k_T^{n-1}| \gg \dots |k_T^1|$.

At sufficiently large x logarithms of $1/x$ necessarily appear. We have actually seen them in the double scaling limit. They must, at some point, spoil the predictivity of the perturbative expansion. One must then identify the regions in integration capable of giving rise to terms of the form $(\alpha_s \log \frac{1}{x})^n$, and eventually to $\alpha_s^n \log^{n-1} \frac{1}{x}$ and so on.

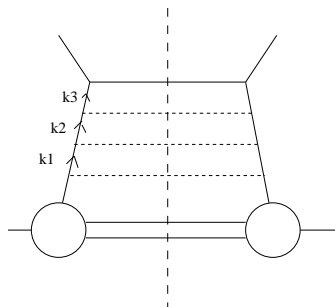


Figure 9: Ordering leading to the most singular $\log 1/x$ contribution.

Lipatov and coworkers[28] (see also[30] for alternative derivations) have identified such a contribution. It corresponds to the diagram depicted in figure 10, more specifically to the region

$$k_i = \alpha_i P + \beta_i n + K_{iT}, \quad (85)$$

$$\alpha_1 \gg \alpha_2 \gg \dots \gg \alpha_{n-1}, \quad k_{iT} \sim k_{jT}, \quad \beta_1 \ll \beta_2 \ll \dots \ll \beta_{n-1}. \quad (86)$$

This leads to splitting kernels similar to those of the Altarelli-Parisi equations

$$F_2(x, Q^2) = \int d^2 k_T \int_x^1 \frac{d\xi}{\xi} C\left(\frac{x}{\xi}, Q^2, k_T\right) F_2(\xi, k_T), \quad (87)$$

where $F_2(\xi, k_T)$ obeys the differential equation

$$\xi \frac{\partial}{\partial \xi} F_2(\xi, k'_T) = \int d^2 k_T K(k'_T, k_T) \frac{k_T'^2}{k_T^2} F_2(\xi, k_T). \quad (88)$$

The BFKL kernel is now known to leading and subleading order.[28, 31] The leading asymptotic solution is

$$F(x, k_T) \sim x^{-4N \log \frac{2\alpha_s}{\pi}}. \quad (89)$$

Unfortunately the corrections implied by the next-to-leading calculations are gigantic[31]. There is no way of doing anything useful with BFKL scaling at present.

As previously discussed this does not seem to be a problem for HERA data since a careful analysis shows that —perhaps surprisingly— the data is well accounted for by ordinary perturbative QCD (the matter is however somewhat controversial to this date), but it will come the day where $\log 1/x$ corrections will be essential. The subject is thus still open.

Acknowledgements

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